Structural Analysis-I

DEGREE OF STATIC AND KINETIEC INDETERMINANCY

Statically Determinate structure

- The structure for which the reactions at the supports & internal forces in the members can be found out by the conditions of static equilibrium, is called a statically determinate structure.
- Example of determinate structures are: **simply supported beams, cantilever beams, single and double overhanging beams, three hinged arches**, etc.
- There are three basic conditions of static equilibrium:

$$\Sigma H = 0$$
$$\Sigma V = 0$$
$$\Sigma M = 0$$

Statically indeterminate structure

- The structure for which the reactions at the supports & the internal forces in the members can not be found out by the conditions of static equilibrium, is called statically indeterminate structure.

- Examples of indeterminate structures are: **fixed beams, continuous beams, fixed arches, two hinged arches, portals, multistoried frames**, etc.

- If equations of static equilibrium are not sufficient to determine all the unknown reactions (vertical, horizontal & moment reactions) acting on the structure, it is called externally indeterminate structure or externally redundant structure.

- If equations of static equilibrium are not sufficient to determine all the internal forces and moments in the member of the structure, even though all the external forces acting on the structure are known, is called internally indeterminate structure or internally redundant structure.

DEGREE OF STATIC INDETERMINCY(D_S):

- The degree of static indeterminacy of the structure may be defined as the number of unknown forces in excess of equations of statics.
  It is also known as degree of redundancy.

Therefore,

degree of static indeterminacy

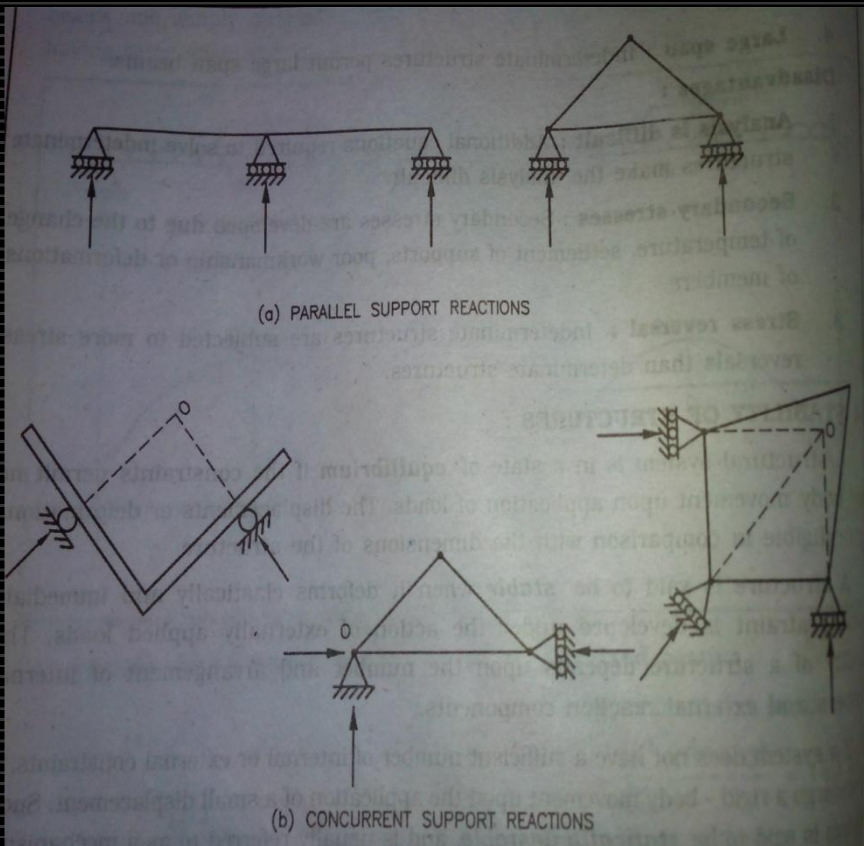
= Total no. of unknown forces – Number of equations of static available

- Redundants may be support reactions or internal forces in the members.
- If redundants are removed from the structure, it becomes determinate.
- Thus, the degree of indetermincy is equal to the number of releases necessary make the structure determinate.

$$D_S = D_{SE} + D_{SI}$$

S. No.	Determinate Structures	Indeterminate Structures
1	Equilibrium conditions are fully adequate to analyse the structure.	Conditions of equilibrium are not adequate to fully analyze the structure.
2	Bending moment or shear force at any section is independent of the material property of the structure.	Bending moment or shear force at any section depends upon the material property.
3	The bending moment or shear force at any section is independent of the cross-section or moment of inertia.	The bending moment or shear force at any section depends upon the cross-section or moment of inertia.
4	Temperature variations do not cause stresses.	Temperature variations cause stresses.
5	No stresses are caused due to lack of fit.	Stresses are caused due to lack of fit.
6	Extra conditions like compatibility of displacements are not required to analyze the structure.	Extra conditions like compatibility of displacements are required to analyze the structure along with the equilibrium equations.

External stability:



- The support system shown in fig. (a) are unstable, because it can provide only parallel reaction components. It cannot, therefore, resist a force perpendicular to the direction of the reactive forces.
- The support system shown in fig. (b) are also unstable, because the reactive forces are concurrent. Hence, the support system is incapable of resisting a couple about the point “o”.

Internal stability:

- A structural system is internally stable if it can preserve its geometry under the action of all kinds of forces tending to deform it. Significant internal forces are produced in the members of a structural system as a result of even small changes in geometry.

- On the other hand, the geometry of unstable system, known as MECHANISMS, can change substantially without generating appreciable internal forces.

- For pin-jointed plane & space structures, if the number of members is less than the minimum requirement, an unstable system, known as mechanism is obtained.

- If number of members is more than the minimum required, an overstiff statically indeterminate system is obtained.

- If the number of members is equal to the minimum required, a stable & determinate system is obtained.

- A rigid-jointed frame is internally stable & statically determinate if it has an open configuration.

DEGREE OF REDUNDANCY OF BEAMS:

- In case of continuous beams, the shear & moment at any point in the beam are readily known once the reaction components are determined. Thus these beams are statically determinate internally.

- Therefore,

  For beams, $D_{Si} = 0$

The degree of indeterminacy of a beam is therefore equal to its external redundancy.

$$D_s = D_{Se}$$

- As stated earlier,

  $DS = DSe = R - r$

  Where, R = total no. of reaction components

  r = total no. of condition equations available

Example for beam:

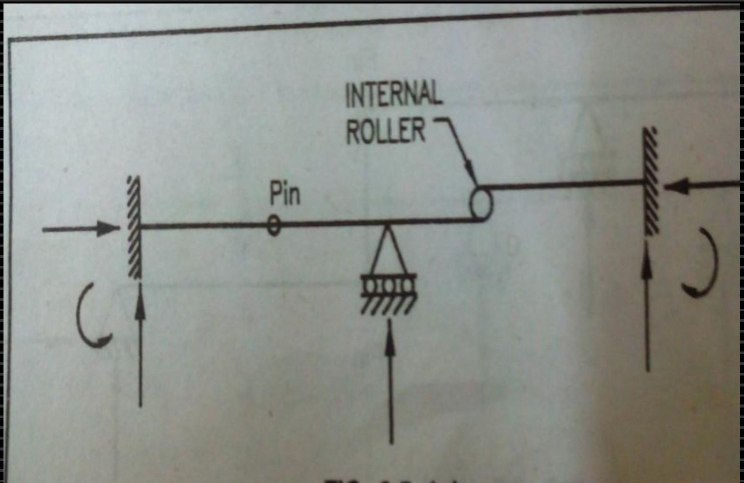


FIG. 3.5 (p)

- $D_S = R - r$

Where, R = No. of reaction components.

r = no. of condition equation available.

- $D_S = (3+1+3) - 6$

  $= 1$

- Therefore, beam is indeterminate to first degree.
- Internal roller act as a link & provide two extra conditions.

DEGREE OF REDUNDANCY OF PLANE TRUSS:

- A truss or an articulated structure, is composed of links or bars, assumed to be connected by frictionless pin s at the joints, and arranged so that the area enclosed within the boundaries of the structure is sub divided by the bars into geometrical figures which are usually triangles.

- If all the members of a truss lie in one plan , it is called a plane truss.

- If members of a truss lie in three dimensions, it is called a space truss.

- In plane truss or space truss loads are applied at the joints only.

- The member of truss are subjected to only axial forces.

- At each joint of a plane truss, two equilibrium equations are available.

  $\Sigma H = 0$ & $\Sigma V = 0$

Difference between truss & frame:

Truss	Frame
A truss is composed of links or straight bars assumed to be connected by frictionless pins at the joints.	A frame is a structure composed of links or straight bars connected at their ends by rigid joints.
Forces are applied only at the joints.	Forces may act any where on the member.
Members are subjected to axial forces only.	Members are subjected to axial forces, shear & moment.
The members of truss experiences deformation in the for of axial compression or tension in nature.	The members of frame have significant deformations in the form of flexural & axial deformations.
The members sub divide the structure into geometrical figures. Which are usually triangles.	The members sub divide the structure into geometrical figures. Which are usually rectangles.

Statically determinate truss:

- A truss in which unknowns can be determined by equilibrium equations alone, is called statically determinate truss.

- The necessary condition for statically determinate truss is,

$$m = 2j - r$$

where, m = no. of members

J = no. of joints

r = no. of condition equations available

Types of trusses:

Types of trusses

↓

Perfect truss

$m = 2j - r$

It is determinate and stable

Imperfect truss

$m = 2j - r$

↓

Deficient truss

$m < 2j - r$

The structure is always unstable

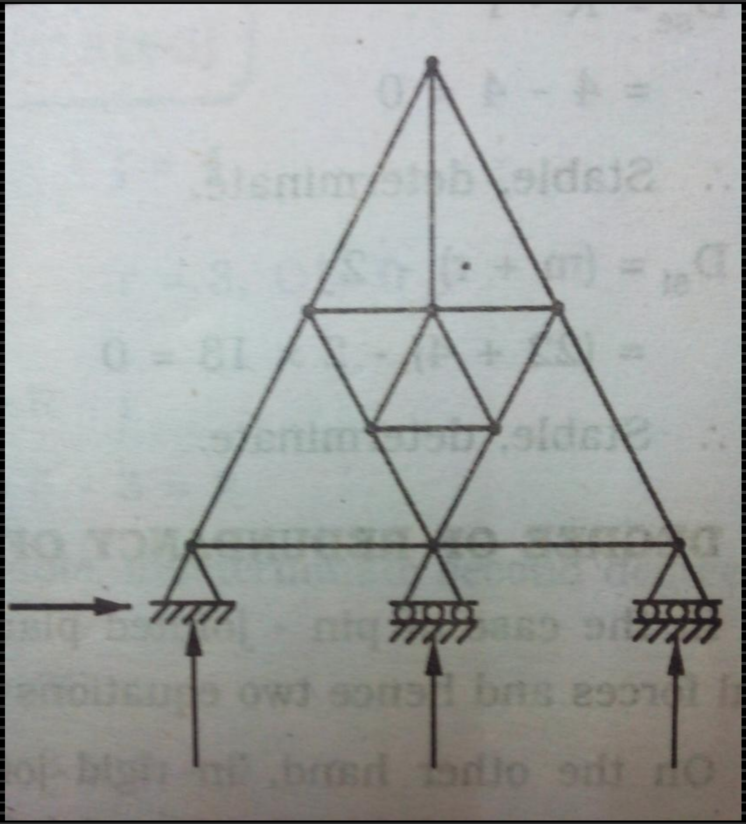
Redundant truss

$m > 2j - r$

It is stable & statically indeterminate

$Dsi = (m+r) - 2j$

Example of plane trusses(pin jointed):



m = 16 J = 9

R = 4 r = 3

- Dse = R – r

  = 4 – 3

  = 1

Therefore, stable, indeterminate first degree.

- Dsi = (m+r) – 2j

  = (16+3) – 2(9) = 1

Therefore, indeterminate first degree.

- Ds = Dse + Dsi

  = 1+1 = 2

Therefore, indeterminate second degree.

DEGREE OF REDUNDANCY OF PLANE FRAMES:

- In the case of pin-jointed plane frames or trusses, the members carry only axial forces & hence two equations are available at each joint.

$$\Sigma H = 0 \quad \& \quad \Sigma V = 0$$

- In rigid jointed plane frame, each member carry three unknown internal forces (i.e. axial force, shear force & B.M.). Therefore, three equations are available at each joint.

- Degree of indeterminacy may be calculated by the following equation, alternatively,

$$D_s = (3m+R) - 3j$$

where, R = total no. of external reaction components.

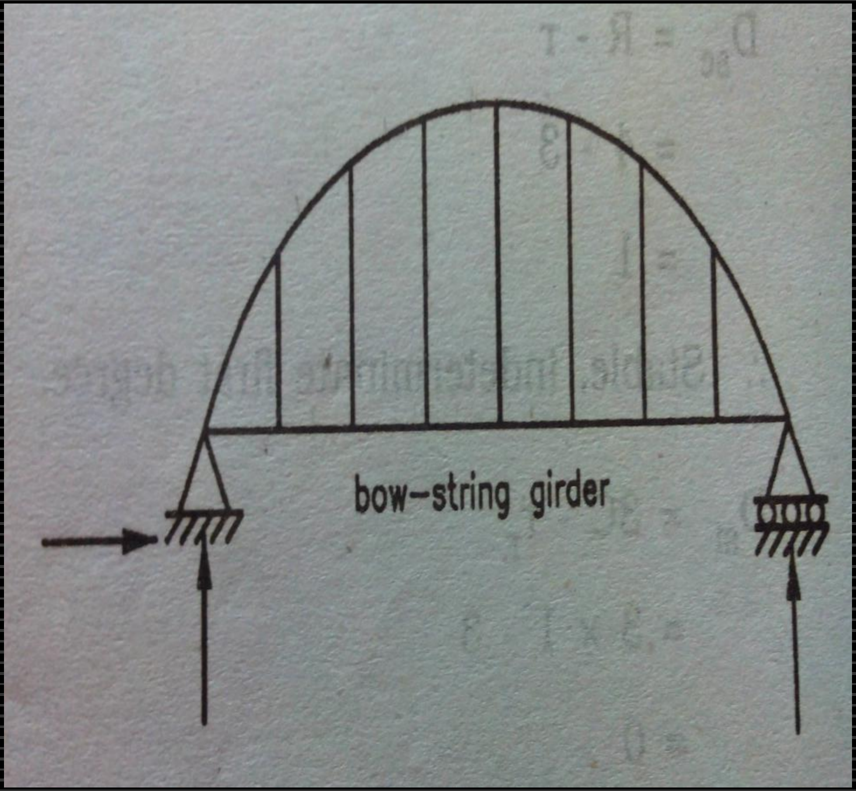
- In case of hybrid structures, i.e. rigid jointed plane frame with few pin joints, the degree of total indeterminacy (DS) may be calculated by the following equation.

$$Ds = (3m - rr) + R - 3(j + j')$$

Where, rr = no. of members connected to pin or hinge

j' = no. of hinges.

Examples of plane frame(Rigid jointed):



(1) m = 23

j = 16

R = 3

r = 3

C = 8

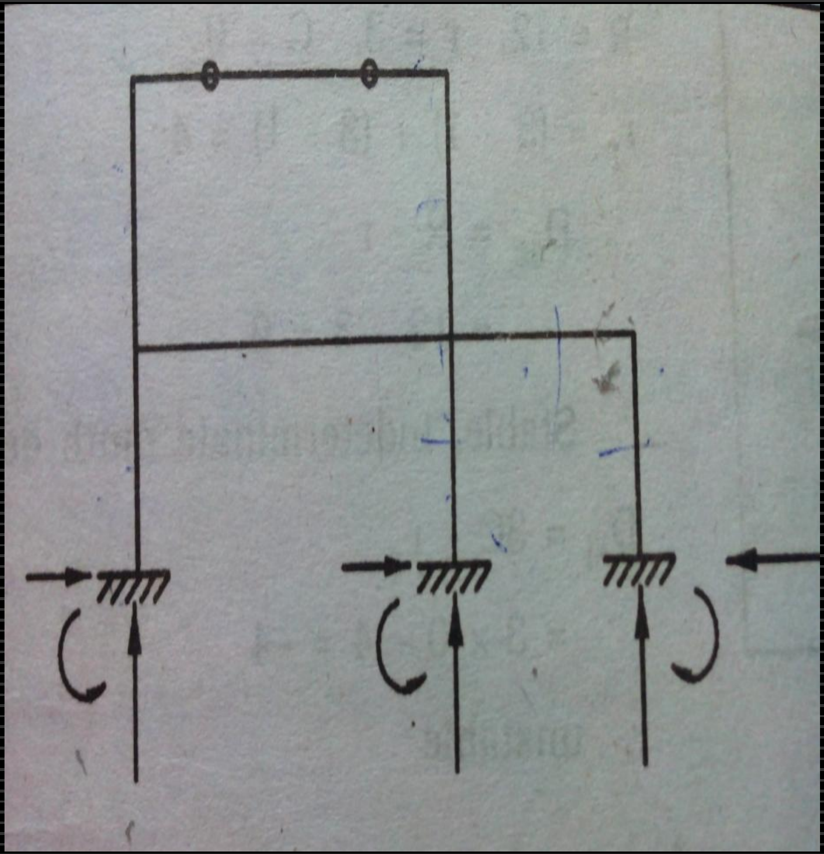
Ds = 3m + R – 3j

= (3 x 23) + 3 – 3 x 16

= 24

Therefore, stable, indeterminate twenty fourth degree.

Plane frame(Rigid jointed):



(2) m = 10

j = 8

j' = 2

R = 9

r = 3

C = 1

DS = (3m – rr) + R – 3(j + j')

= (3 x 10 – 2) + 9 – 3(8 + 2)

= 28 + 9 – 30

= 7

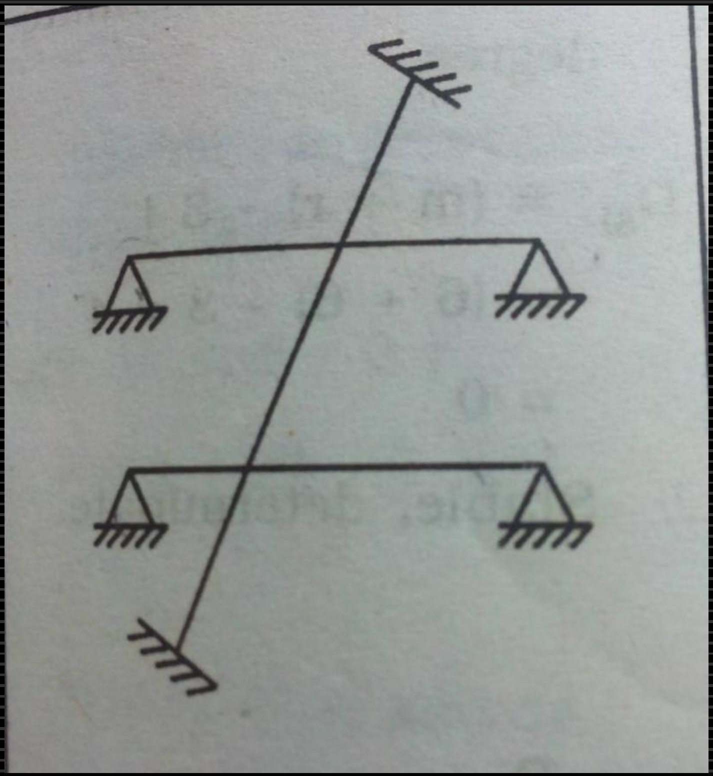
Therefore, stable indeterminate seventh degree.

DEGREE OF REDUNDANCY OF GRIDS:

- A grid is a plane structure composed of continuous members that either intersect or cross each other.
- All members of a grid, generally lie in one plan.
- The members of grid are subjected to vertical shear force, bending moment & twisting moment at any cross section.
- Degree of static indeterminacy for grid is given by,

$$D_s = 3m + R - 3j$$

Example for grids:



m = 7

J = 8

R = 14

Therefore.

DS = 3m + R – 3j

= (3 x 7) + 14 – 3 x 8

= 11

Stable, indeterminate eleventh degree.

DEGREE OF REDUNDANCY OF SPACE TRUSS:

- In the case of space truss, all the members of the truss do not lie in one plan. Very often , space truss is formed by combining a series of plane trusses.
- The members of a space truss are subjected to axial forces only.
- The equilibrium of an entire space truss or sections of a space truss is described by the six scalar equations given below.

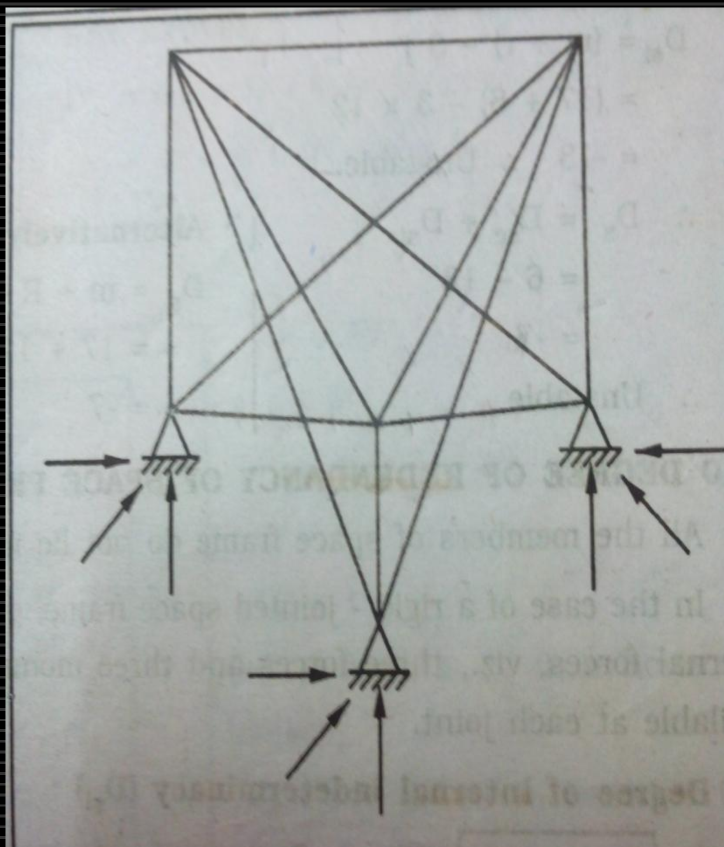
$$\Sigma F_X = 0, \quad \Sigma F_Y = 0, \quad \Sigma F_Z = 0$$

$$\Sigma M_X = 0, \quad \Sigma M_Y = 0, \quad \Sigma M_Z = 0$$

- Degree of static indeterminacy for space truss is given by,

$$D_S = m + R - 3j$$

Example of space truss:



m = 12

J = 6

R = 3 x 3 = 9

r = 6

Ds = m + R – 3j

= 12 + 9 – 3 x 6

= 3

Therefore, stable, indeterminate third degree.

DEGREE OF REDUNDANCY OF SPACE FRAMES : (RIGID- JOINTED)

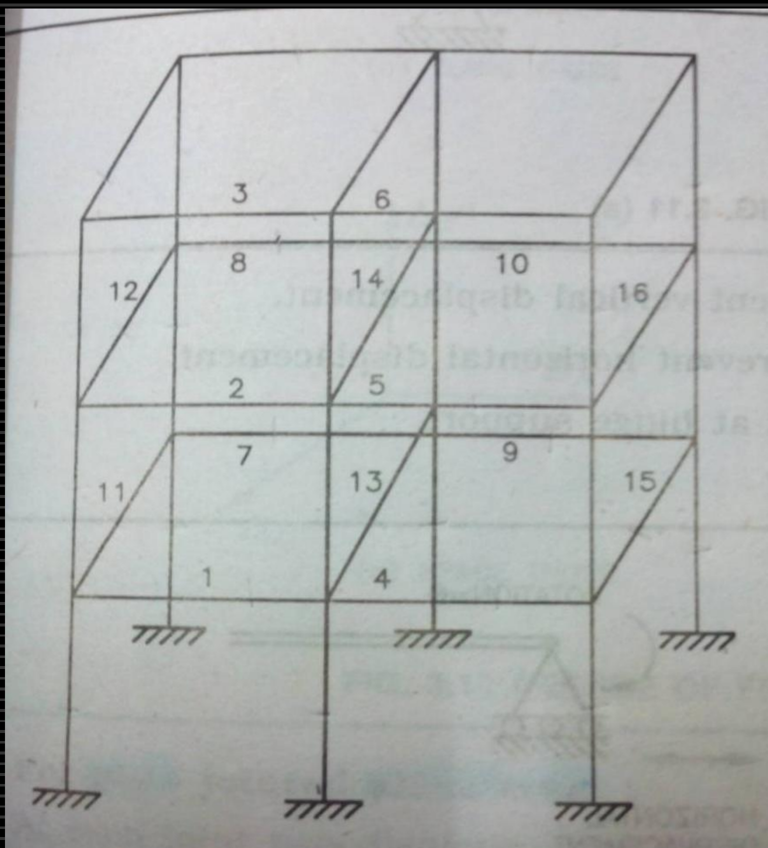
- All the members in space frame do not lie in one plane.

- In the case of a rigid – jointed space frame, every member carries six unknown internal forces, three forces & three moments. Therefore, six equations are available at each joint.

- Degree of static indeterminacy for space frames is given by,

$$D_s = 6m + R - 6j$$

Example of space frame(Rigid jointed):



m = 39

J = 24

R = 6 x 6 = 36

r = 6

C = 16

Dse = 6m + R – 6j

= (6 x 39) + 36 – 6 x 24

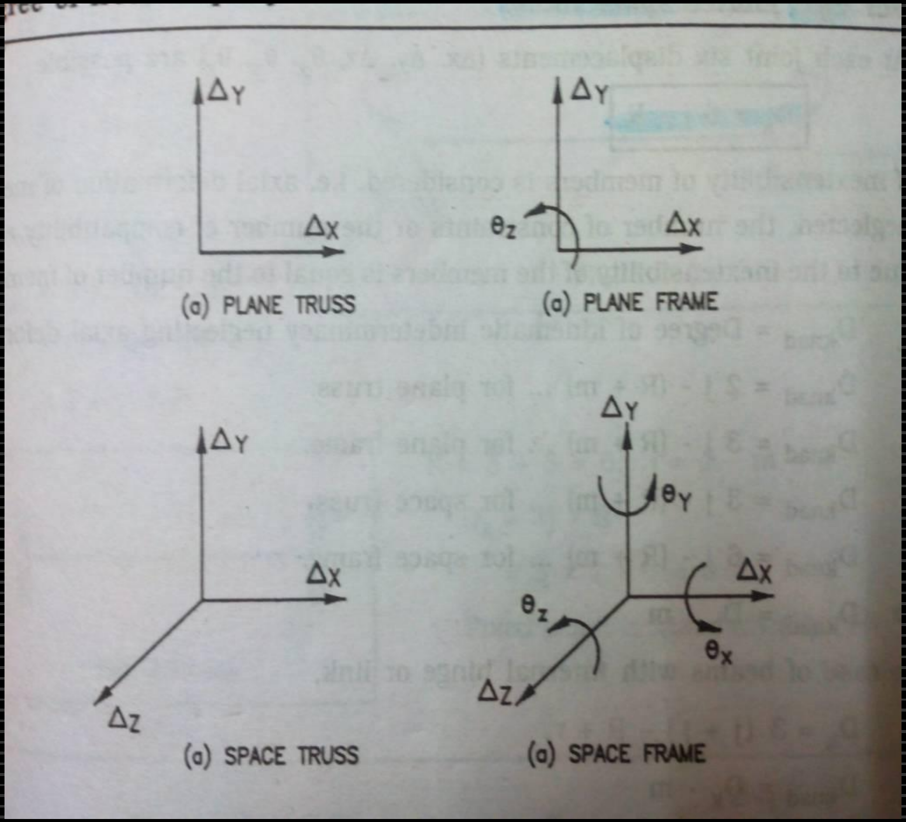
= 126

DEGREE OF KINEMATIC INDETERMINACY OR DEGREE OF FREEDOM:

- If the displacement components of the joints of a structure cannot be determined by compatibility equations alone, the structure is said to be <u>kinematically indeterminate structure.</u>

- The no. of additional equations necessary for the determination of all the independent displacement components is known as the <u>degree of kinematic indeterminacy</u> or the <u>degree of freedom</u> of the structure.

- Thus degree of kinematic indeterminacy is equal to the total no. of possible displacements in a structure.



Kinematic Indeterminacy for diff. trusses & frames:

- For pin-jointed plane truss:

  Dk = 2j – r

- For Pin-jointed space truss:

  Dk = 3j – r

  At each joint three displacements are possible.

- For Rigid-jointed plane frame:

  Dk = 3j – r

  At each joint three displacements are possible.

- For Rigid-jointed space frame:

  Dk = 6j - r

- If inextensibility of members is considered i.e. axial deformation of members is neglected, the no. of constraints or the no. of compatibility equations due to inextensibility of the members is equal to the no. of members.

D_{kand} = Degree of kinematic indeterminacy neglecting axial deformations.

$D_{kand} = 2j - (R + m)$ … for plane truss

$D_{kand} = 3j - (R + m)$ … for space truss

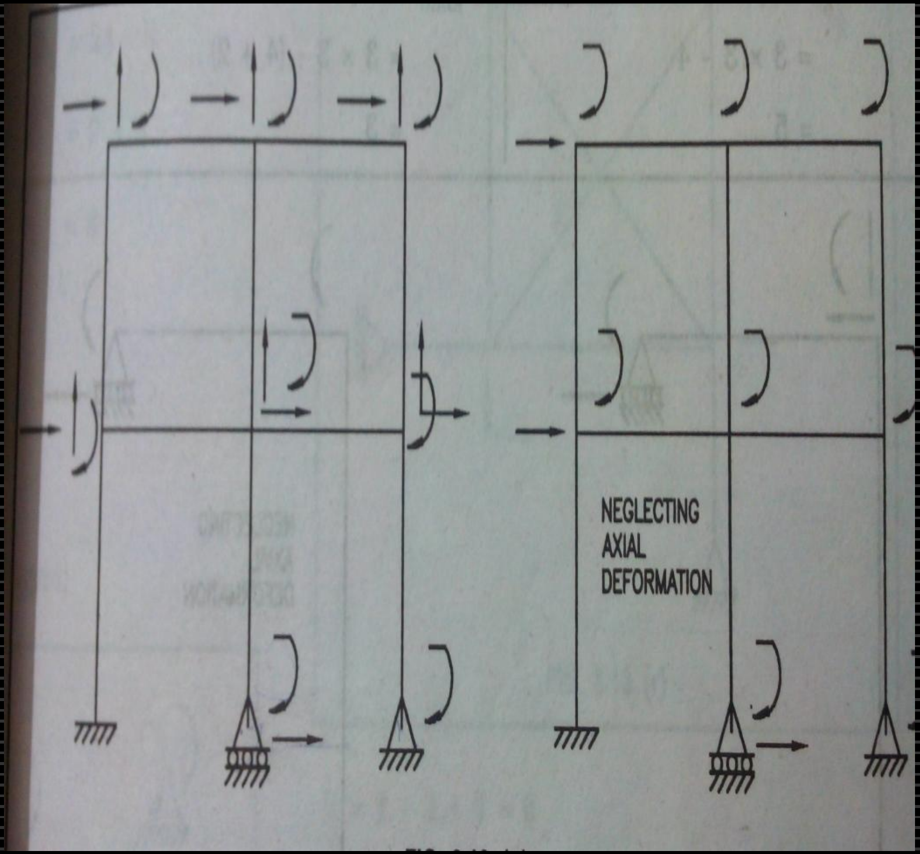
$D_{kand} = 3j - (R + m)$ … for plane frame

$D_{kand} = 6j - (R + m)$ … for space frame

- In case of beams & plane frame with internal hinge or link:

$$D_k = 3(j + j') - R + rr$$

Example:



R = 3 + 1 + 2 = 6

m = 10

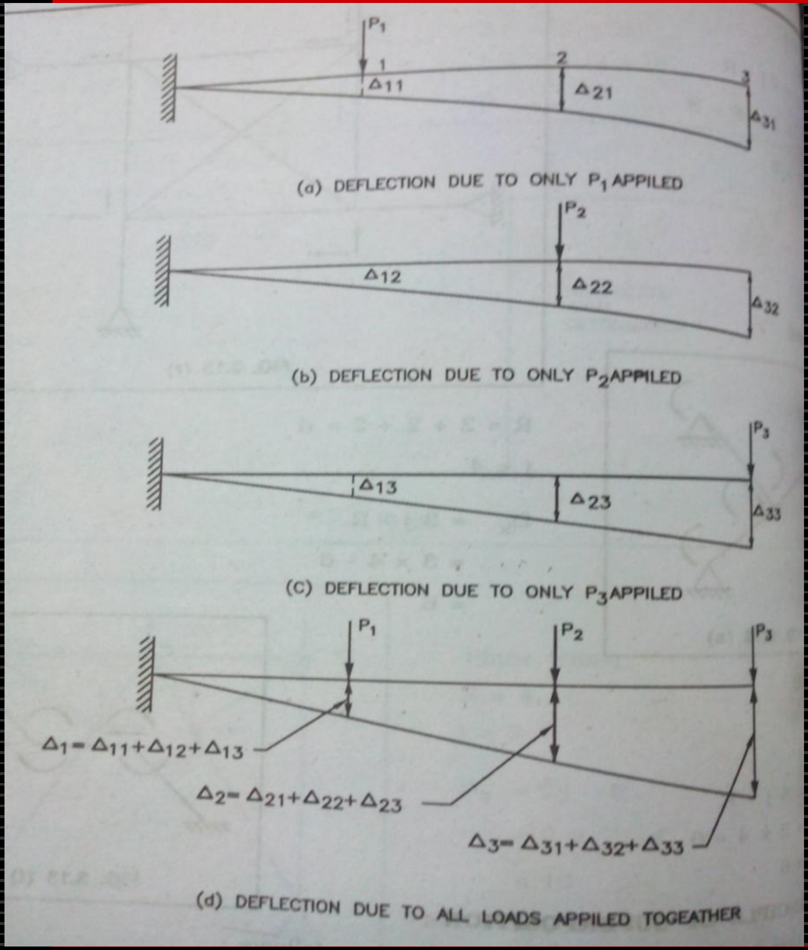
J = 9

Dkand = 3j – (R + m)

= 3 x 9 – (6 + 10)

= 11

PRINCIPLE OF SUPERPOSITION:



- Super position allows us to separate the loads in any desired way, analyse the structure for a separate set of loads & find the result for the sum of loads by adding individual load effects.
- Superposition applies equally to forces , stresses, strains & displacements.